Structures & type classes

$\sum_{(x:A)} B(x)$ elements (x,y) where $x:A$
$y:B(x)$

$B : A \to \mathrm{Type}\ u$

Structure:

- points in the plane
- group, ring
- mathematical structures: topol. spaces, manifolds, graphs, ...

$$\sum_{(x:A)} \sum_{(y:B(x))} \sum_{(z:C(x,y))} \cdots \sum_{\cdots} T(x,y,z,\ldots)$$

element $(x,(y,(z,\ldots(\ldots,t))\ldots)$

record $\{ l_1 := x,\ l_2 := \_\ ,\ \ l_n := \ldots \}$

name the components

$\mathbb{R}^{\{0,1\}}$ $\mathbb{R}^{\{re,\,im\}}$

Non-dependent: $A \times B \times C \times \cdots \times T$

Extensionality principle:

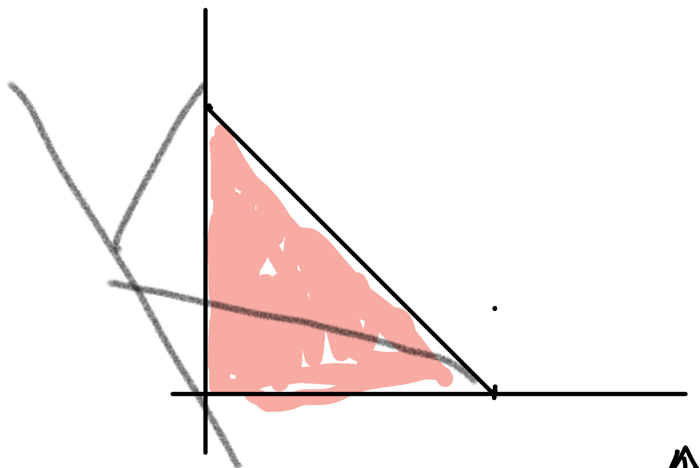
$u, v : A \times B$

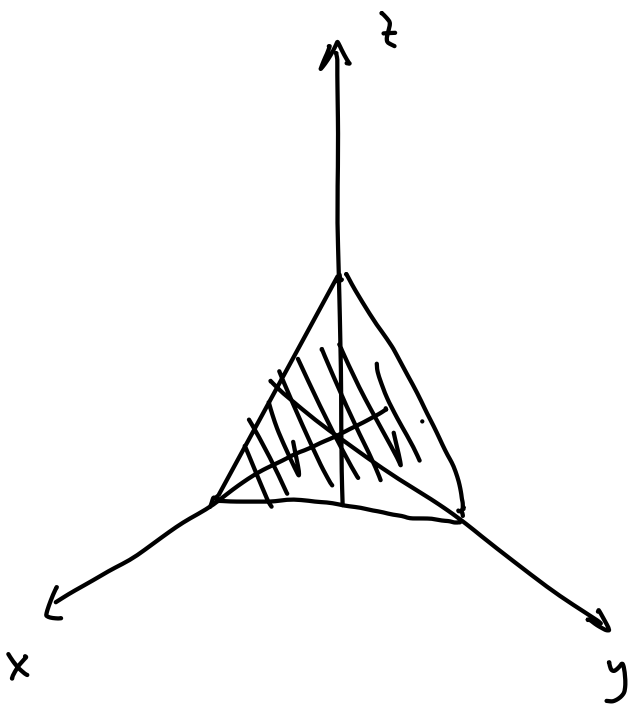
$$\pi_1 u = \pi_1 v \Rightarrow \pi_2 u = \pi_2 v \Rightarrow u = v$$

$$\forall (\varphi : \mathrm{Prop})\ (p, q : \varphi).\ p = q$$

$$f, g : A \to B : \quad (\forall (x : A),\ fx = gx) \Rightarrow f = g$$

funext





$$\mathbb{R} \times \underline{\mathbb{R} \times \mathbb{R}}$$

n-simple...

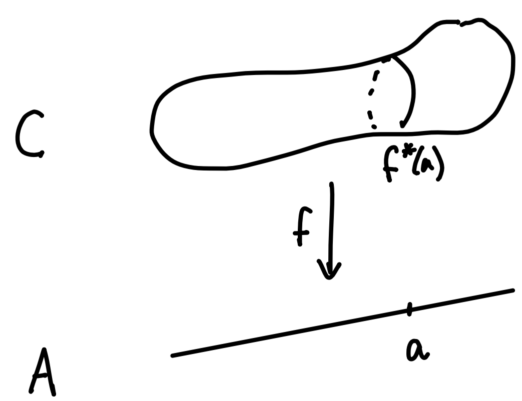
$$\begin{aligned} \mathbb{N} &\longrightarrow \mathrm{Type} \\ n &\longmapsto \text{structure} \ldots\ldots \end{aligned}$$

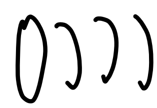
Display maps vs. fibrations

$$\mathrm{Set}/_A \simeq \mathrm{Set}^A$$

$$\begin{array}{ccc} C & & \\ f \downarrow & \longmapsto & (a : A) \mapsto f^*(a) = \{c : C \mid f c = a\} \\ A & & C_a \end{array}$$

$$\begin{array}{ccc} \sum_{(a:A)} B(a) & & \\ \pi_1 \downarrow & \longleftarrow\!\!| & B : A \to \mathrm{Set} \\ A & & \end{array}$$





Arend

Structure
- $\alpha : \mathrm{Type}_*$
- mul : . . . .
- one :
- inv :

$$\mathrm{Structure} \xrightarrow{\alpha} \mathrm{Type}_*$$

$\mathrm{Group} : \mathrm{Type}_* \to \mathrm{Type}_*$

$\alpha \mapsto$ structure
- mul
- one
- inv
- ⋮

Monoid

AbGroup   Monoid

Ring

How to organize <u>hierarchies</u>?

? { How to know "the implied" structure not referred to explicitly?

1) Basic case: "the" group structure on $\mathbb{Z}$
2) Infer the structure: A, B sets which carry some unspoken group str.

   $A \times B$

(type) classes

Monoid α

$$\begin{tikzcd}[column sep=small]
 & \text{Group } \alpha \arrow[dl, dash] & & \\
\underset{\text{abelian}}{\text{AbGroup } \alpha} & & & \overset{1,\cdot}{\text{Monoid } \alpha} \\
 & \underset{\text{distr}}{\text{Ring } \alpha} \arrow[ul] \arrow[urr] & &
\end{tikzcd}$$

$$\begin{tikzcd}[column sep=small]
 & D & \\
A \arrow[ur, dash] & & B \arrow[ul, dash] \\
 & C \arrow[ul] \arrow[ur] &
\end{tikzcd}
\quad ? \quad
\begin{tikzcd}[column sep=small]
D & & D \\
A \arrow[u] & & B \arrow[u] \\
 & C \arrow[ul, dash] \arrow[ur, dash] &
\end{tikzcd}$$

Reconstruct missing info:

forced

→ implicit arguments $\{\ldots\ldots\}$

→ Class $[\ldots\ldots]$

rules on how to get this